Find the derivative:

① $f(t) = t^2 \arcsin(t)$

② $g(x) = 2x\ln(x)\cdot e^x$

③ Find $\ln(1+4\cdot 10^{-30})$, accurate to 8 decimal places.

① $f'(t) = \boxed{2t\arcsin(t) + t^2\frac{1}{\sqrt{1-t^2}}}$

② $g'(x) = 2\ln(x)e^x + 2x\frac{1}{x}\cdot e^x + 2x\ln(x)e^x$

$= \boxed{2\ln(x)e^x + 2e^x + 2x\ln(x)e^x}$

$= 2e^x(\ln(x) + 1 + x\ln(x))$.

$\boxed{\ln(A\cdot B) = \ln(A) + \ln(B)}$.

③ $\lim\limits_{h\to 0} \frac{\ln(1+h) - \ln(1)}{h} = \frac{d}{dx}\left(\ln(x)\right)\Big|_{x=1} = \frac{1}{x}\Big|_{x=1}$

$= 1$

$\frac{\ln(1+4\cdot 10^{-30}) - \overset{0}{\ln(1)}}{4\cdot 10^{-30}} \approx 1$

$$\Rightarrow \ln(1+4\cdot 10^{-30}) \approx 4\cdot 10^{-30}$$

$$= \approx \underbrace{.000\cdots0}_{29 \text{ zeros}}4000\cdots$$

New topic:

Applications of the derivative to real life situations.

Related Rates ~ When an equation relate two variables, we can relate their derivatives (usually with respect to time).

Example $A = s^2$

Suppose both $A$ & $s$ are functions of $t$ (time).

If $s = 5$ & $\frac{ds}{dt} = 3$, what's $\frac{dA}{dt}$?

Solution: $A = s^2$

Differentiate: $\frac{dA}{dt} = \frac{d}{dt}(s^2) = 2s\cdot\frac{ds}{dt}$

$$\Rightarrow \frac{dA}{dt} = 2(5)\cdot(3) = \boxed{30}.$$

Example of a question we just answered -

A square is increasing in size. Each of its sides are increasing at a rate of 3 in/s when the side length is 5 inches. How fast is the area of the square increasing?


$\boxed{\text{Want } \frac{dA}{dt}}$. $\boxed{A = s^2}$

$$\Rightarrow \frac{dA}{dt} = \frac{d}{dt}(s^2) = 2s\frac{ds}{dt}$$
$$= 2 \cdot 5\,(3) = \boxed{30 \, \frac{in^2}{s}}$$

Example Curran's zamboni is traveling north on Lake Michigan, going 75mph. Meanwhile Austin drives his zamboni, moving west at a rate of 20mph. If they both start at the same point, how fast is the distance between them increasing after 30 minutes?

Guesses

H: 20mph

J: 60mph

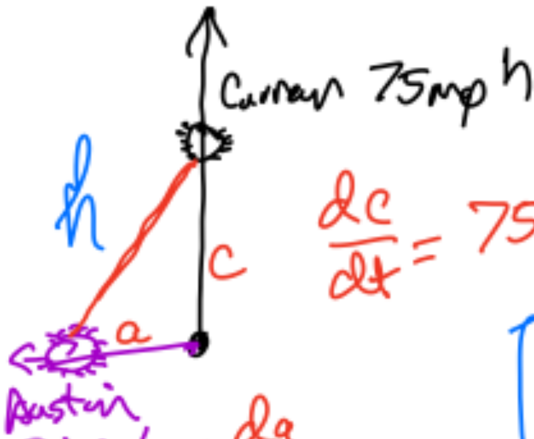
R: 120mph

Aman: 2mph

A: 25mph

B: 40mph

K: 48mph




20mph $= \frac{da}{dt}$

$\frac{dc}{dt} = 75\text{mph}$

$\boxed{\text{Want } \frac{dh}{dt}}$

$$h^2 = a^2 + c^2$$

differentiate:

$$2h\frac{dh}{dt} = 2a\underset{\uparrow\ 20\text{mph}}{\frac{da}{dt}} + 2c\underset{\uparrow\ 75\text{mph}}{\frac{dc}{dt}}$$

after $\frac{1}{2}$ hour:
(from question)

$$c = \frac{75}{2} = 37.5 \text{ miles}$$

$$a = \frac{20}{2} = 10 \text{ miles}$$

$$h = \sqrt{a^2 + c^2} = \sqrt{100 + (37.5)^2}$$
$$= \sqrt{1506.25}$$
$$= 38.81 \text{ miles}$$

$$2(38.81 \text{ miles})\frac{dh}{dt} = \underbrace{2(10 \text{ miles})(20\text{mph})}_{400} + \underbrace{2(37.5 \text{ miles})(75\text{mph})}_{5{,}625}$$

$$77.62\frac{dh}{dt} = 6025$$

$$\frac{dh}{dt} = \frac{6025}{77.62} = \boxed{77.62\text{ mph}}$$